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A COLLOCATION METHOD FOR CONVECTION DOMINATED FLOWS

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SUMMARY

A collocation method based on multiple regions with moving boundaries placed in a flow field in which convection effects dominate, is proposed. By making the moving boundaries of the regions coincide with moving sharp fronts present in the solution of convection dominated problems, and thereby allowing higher concentration of meshes to be placed about the fronts, the proposed method is able to achieve very high accuracy. By having a moving mesh, the Peclet number characterizing the flow field depends upon velocity relative to a moving mesh in a region. Consequently by choosing proper velocities of the moving boundaries, the value of this Peclet number can be made as small as desired. The traditional collocation method based on centred discretization, when applied to each region in the field, produces oscillation free solutions even when the values of Peclet number based on absolute velocity are extremely large. In view of these characteristics the method appears to be an excellent candidate for the solution of any two-phase flow problem containing sharp fronts.

KEY WORDS Collocation Moving Mesh Advection–Diffusion

INTRODUCTION

Spline collocation schemes with optimum choice of collocation points have been very successful in obtaining very accurate solutions to diffusion dominated problems or problems in which diffusion is either greater, or of the same order as, convection.[1–6] For convection-dominated flows with single-step time differencing (such as the Crank–Nicholson scheme) collocation schemes, like other numerical approaches (such as central finite-differencing[7] and finite elements methods,[8]) lead to oscillatory behaviour.[9] The present investigation shows that a collocation scheme with multistep time-differencing appears to yield oscillation free solutions for convection-dominated flows for significantly higher mesh Peclet numbers (P_Δ) than has been possible previously with single-step time differencing.[9] The stability limit is considerably higher than $P_\Delta = 2$, which is typical of values for central finite-difference schemes. The present computations for the model diffusion problem using multistep time differencing show that for P_Δ even up to 100, there appear to be no noticeable oscillations in the solution. However, as P_Δ approaches higher values, the oscillations begin to appear.

A number of alternative schemes have been proposed to remove these oscillations. Almost all of these rely on upstream weighting of the convection terms, for example the upstream

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differencing of the convection term in the finite-difference method, and the use of asymmetric basis functions for collocation and finite-element methods.[9,10] These schemes are not very satisfactory as they tend to suppress oscillations by introducing artificial numerical diffusion and consequently they suffer from severe inaccuracies, particularly in problems which contain sharp moving fronts such as those due to the presence of shock waves, phase change or heterogeneous multiphase mixtures.

In this paper a method is proposed to circumvent the problem of oscillations by requiring the effective mesh Peclet number to be small, or making it approach zero. In the latter case, the problem effectively reduces to a pure diffusion problem. This is accomplished by dividing up the flow field into two or more regions. The choice of the actual number of regions will depend upon the heterogeneity of the media, and the level of variation of the fluid velocity and of various other dependent variables. If the boundaries of these regions are allowed to move relative to some fixed boundary with the local fluid or front velocity, then meshes placed in these regions also have motion relative to the fixed boundary. As a consequence of this, the velocity of the flow field relative to the moving mesh can be reduced to a very small value or can be made to approach zero so that the effective Peclet number based on this relative velocity can be made as small as necessary. The governing equations to be solved are transformed to these moving co-ordinates, the relevant mesh Peclet number is then based on fluid velocity relative to the local velocity of the mesh. Consequently this mesh Peclet number can be made as small as necessary to circumvent the appearance of undesirable oscillations and still be able to achieve the high order accuracy which is obtainable from centred discretization resulting from the use of a spline collocation scheme. This method is an extension of the moving co-ordinate method originally devised by O'Neill and Lynch[8] for the solution of convective-diffusion problems using a finite-element method based on the Galerkin procedure.

A similar approach was pursued in a paper by Jensen and Finlayson.[11]* These authors have considered the same problem using a moving mesh procedure with C^1 cubic elements in an orthogonal collocation scheme and Crank–Nicholson time differencing. The present treatment differs from their work in two respects: (1) We have introduced the concept of a multiregional moving mesh scheme which not only allows the tracking of moving fronts in the media but can also deal with heterogeneity of the media, such as that which occurs in the presence of multiphase flows. (2) In addition, we have used multistep time differencing based on the so-called backward differentiation formulae as incorporated in the ODE solver package LSODI.[12] This scheme in terms of accuracy, as already pointed out, has a significant advantage over the single-step Crank–Nicholson technique. This difference will be further highlighted by actual numerical examples.

Related papers by Miller and Miller[13] and by Miller[14] also use moving co-ordinates, but are based on a finite-element method using the Galerkin procedure. The principal drawback of finite element methods based on the Galerkin or a weighted residual procedure is that these methods require evaluation of integrals by numerical quadrature at each time step and thus entail considerably more arithmetic, especially for a non-linear problem, as compared to a collocation method.

FORMULATION OF THE METHOD

Let us consider a general one-dimensional flow field as shown in Figure 1. Let $\phi(x, t)$ be the dependent variable, such as concentration or temperature, containing sharp 'fronts' at a

* The existence of this paper was pointed out to us by one of the referees, to whom we are very grateful.

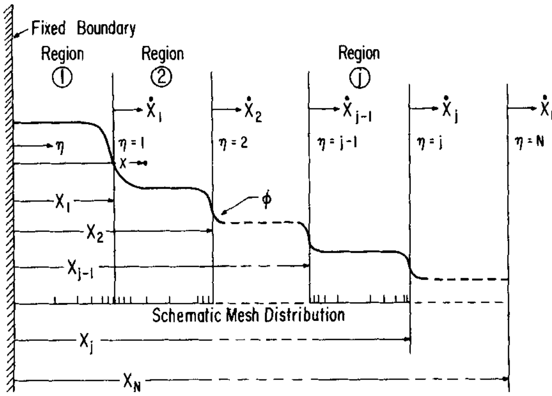

Figure 1. Moving mesh co-ordinate system

number of locations in the field. Let us further assume that these fronts are moving to the right in the flow field with the velocity U of the flow. It is clear that computation of ϕ with the conventional non-moving fixed grids would not be able to track moving fronts with any degree of satisfaction except with grid sizes which greatly exceed those that are theoretically necessary for normal engineering accuracy. To circumvent this difficulty, we divide the field into a number of regions. The boundaries of these regions coincide with the fronts in the distribution of ϕ by allowing these boundaries to move with the velocities of these fronts. If a co-ordinate system is established relative to these moving boundaries of the regions, it is then possible to choose the distribution of mesh points such that it has a higher concentration about the fronts and is dispersively distributed elsewhere. Another principal advantage of this co-ordinate system, as we demonstrate below, is that if the equation governing ϕ is expressed in this co-ordinate system the significant Peclet number is based on the velocity relative to the moving mesh rather than the absolute velocity U.

Let x denote the co-ordinate relative to the fixed boundary at the left, as shown in Figure 1, and let η^j denote the co-ordinate in the jth region, relative to the left moving boundary of the corresponding region, which is related to the fixed co-ordinate system through the expression

$$\eta^j = \frac{x - jX_{j-1} + (j-1)X_j}{X_j - X_{j-1}} \quad \text{for} \quad 1 \leq j \leq N \tag{1}$$

where X_j is the non-dimensional (non-dimensionalized with reference length L) co-ordinate of the right boundary of the jth region.

Clearly, the transformation implies that in this relative co-ordinate system, the left boundary always lies at $\eta^j = j-1$ and the right boundary at $\eta^j = j$. This transformation consequently immobilizes the boundaries of the region and, in turn, allows us to choose any desired distribution of the mesh points in a region. The dependent variable ϕ can be written as

$$\phi(x, t) = \phi(\eta^j(x, t), t) \tag{2}$$

which together with equation (1) yields

$$\left(\frac{\partial \phi}{\partial t}\right)_x = \left(\frac{\partial \phi}{\partial t}\right)_{\eta^j} + \frac{\partial \eta^j}{\partial t}\left(\frac{\partial \phi}{\partial \eta^j}\right)_t$$

$$= \frac{\partial \phi}{\partial t} - \frac{j\dot{X}_{j-1} - (j-1)\dot{X}_j + \eta^j(\dot{X}_j - \dot{X}_{j-1})}{X_j - X_{j-1}} \frac{\partial \phi}{\partial \eta^j} \tag{3}$$

$$\left(\frac{\partial \phi}{\partial x}\right)_t = \frac{1}{X_j - X_{j-1}} \frac{\partial \phi}{\partial \eta^j} \tag{4}$$

$$\left(\frac{\partial^2 \phi}{\partial x^2}\right)_t = \frac{1}{(X_j - X_{j-1})^2} \frac{\partial^2 \phi}{\partial \eta^{j2}} \tag{5}$$

Let us now consider a model convective-diffusion problem described by the following equations

$$\frac{\partial \phi}{\partial t^*} + U\frac{\partial \phi}{\partial x^*} = D\frac{\partial^2 \phi}{\partial x^{*2}} \tag{6a}$$

$$\phi(0, t^*) = 1, \qquad t^* \geqslant 0 \tag{6b}$$

$$\phi(x^*, 0) = 0, \qquad 0 < x^* \leqslant L \tag{6c}$$

$$\frac{\partial \phi}{\partial x^*}(L, t^*) = 0, \qquad t^* \geqslant 0 \tag{6d}$$

where U is the velocity, D is the diffusion coefficient, x^* is the dimensional co-ordinate from the fixed origin and t^* is dimensional time. Non-dimensionalization of equation (6a) yields

$$\frac{\partial \phi}{\partial t} + Pe\frac{\partial \phi}{\partial x} = \frac{\partial^2 \phi}{\partial x^2} \tag{7}$$

where $t = t^*D/L^2$ and $x = x^*/L$, $Pe = UL/D$.

The use of equations (3)–(5) in equation (7) yields

$$\frac{\partial \phi}{\partial t} + \frac{Pe - [j\dot{X}_{j-1} - (j-1)\dot{X}_j + \eta^j(\dot{X}_j - \dot{X}_{j-1})]}{X_j - X_{j-1}} \frac{\partial \phi}{\partial \eta^j} = \frac{1}{(X_j - X_{j-1})^2} \frac{\partial^2 \phi}{\phi \eta^{j2}} \tag{8a}$$

which can be written as

$$\frac{\partial \phi}{\partial \tau} + \frac{1}{X_j - X_{j-1}} Pe_r \frac{\partial \phi}{\partial \eta^j} = \frac{1}{(X_j - X_{j-1})^2} \frac{\partial^2 \phi}{\partial \eta^{j2}} \tag{8b}$$

where

$$Pe_r = \frac{U_e L}{D} = Pe - [j\dot{X}_{j-1} - (j-1)\dot{X}_j + \eta^j(\dot{X}_j - \dot{X}_{j-1})] \tag{9}$$

U_e is the velocity of fluid relative to a moving point with co-ordinate η^j. The Peclet number, Pe_r based on this relative velocity can be made as small as desired by making a judicious choice for the velocity of the boundaries of a region. For example if $\dot{X}_j = \dot{X}_{j-1} = Pe$, then the Peclet number, Pe_r for $j > 1$ becomes zero, and convective-diffusion problem (8b) reduces to a pure diffusion problem. As discussed previously, ample experience has accumulated with regard to the use of the collocation scheme with Gaussian quadrature points as collocation points for solving diffusion-type problems and it is well known that it provides very accurate solutions. As will be demonstrated subsequently, this scheme, in combination with multistep time-differencing provides more accurate solutions than other schemes, even at substantially large values of Peclet number. However, there is some overhead which results from the use of variable time steps and local truncation error control.

Equation (8b) applies to the jth region. Across the boundaries between the regions, we require continuity of the values and of the fluxes of the dependent variable ϕ. These conditions yield

$$\phi(X_j^-, t) = \phi(X_j^+, t) \tag{10a}$$

$$D\frac{\partial \phi}{\partial x}\bigg|_{X_j^-} = D\frac{\partial \phi}{\partial x}\bigg|_{X_j^+} \tag{10b}$$

For later use in the collocation method, we require the time derivatives of equations (6b), (10) and (6d) and rewrite them as

$$\frac{\partial \phi}{\partial \tau}(0, t) = \dot{\phi}(0, t) = 0 \tag{11}$$

$$\dot{\phi}(\eta^j = j^-, t) - \dot{\phi}(\eta^{j+1} = j^+, t) = 0 \tag{12}$$

$$\frac{\partial}{\partial t}\left(D\frac{\partial \phi}{\partial x}\bigg|_{X_j^-}\right) - \frac{\partial}{\partial t}\left(D\frac{\partial \phi}{\partial x}\bigg|_{X_j^+}\right) = 0 \tag{13}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial x}(X_N, t)\right) = 0 \tag{14}$$

Transforming equations (13) and (14) to co-ordinate system η^j, we obtain

$$\begin{aligned}\dot{\phi}'(j^-, t) - \frac{X_j - X_{j-1}}{X_{j+1} - X_j}\dot{\phi}'(j^+, t) = {} & \frac{\dot{X}_j - \dot{X}_{j-1}}{X_{j+1} - X_j}\phi'(j^+, t) \\ & - \frac{\dot{X}_{j+1} - \dot{X}_j}{X_{j+1} - X_j}\phi'(j^-, t)\end{aligned} \tag{15}$$

$$\dot{\phi}'(N, t) = 0 \tag{16}$$

where the prime denotes a derivative with respect to η.

COLLOCATION METHOD

We shall seek an approximate solution of equations (8b), (11), (12), (15) and (16) by a collocation method, using cubic Hermite spline basis functions as approximating functions in the spatial variable η^j for $\phi(\eta^j, t)$. More specifically, let the interval $[j-1, j]$ be divided by a set of points, called break points as

$$j - 1 = \eta_1 < \eta_2 < \ldots < \eta_{l+1} = j, \qquad h_i = \eta_i - \eta_{i-1}$$

where we have omitted the use of superscript j for convenience and will restore its use when necessary for clarity. A convenient basis for generating Hermite splines is the set $\{V_i(\eta), S_i(\eta)\}_{i=1}^{l+1}$ where

$$V_i(\eta) = \begin{cases} 3\left\{\dfrac{\eta - \eta_{i-1}}{h_i}\right\}^2 - 2\left(\dfrac{\eta - \eta_{i-1}}{h_i}\right)^3, & \text{for} \quad \eta_{i-1} \leq \eta \leq \eta_i \\ \left(1 - \dfrac{\eta - \eta_i}{h_{i+1}}\right)^2\left[1 + 2\left(\dfrac{\eta - \eta_i}{h_{i+1}}\right)\right], & \text{for} \quad \eta_i \leq \eta \leq \eta_{i+1} \\ 0, \quad \text{otherwise} \end{cases} \tag{17a}$$

$$S_i(\eta) = \begin{cases} h_i\left[-\left(\dfrac{\eta - \eta_{i-1}}{h_i}\right)^2 + \left(\dfrac{\eta - \eta_{i-1}}{h_i}\right)^3\right] & \text{for} \quad \eta_{i-1} \leq \eta \leq \eta_i \\ h_{i+1}\left(\dfrac{\eta - \eta_i}{h_{i+1}}\right)\left(1 - \dfrac{\eta - \eta_i}{h_{i+1}}\right)^2 & \text{for} \quad \eta_i \leq \eta \leq \eta_{i+1} \\ 0, \quad \text{otherwise} \end{cases} \tag{17b}$$

It is assumed that the function $V_1(\eta)$ and $S_1(\eta)$ vanish to the left of η_1 and functions $V_{l+1}(\eta)$ and $S_{l+1}(\eta)$ vanish to the right of η_{l+1}. In addition, we note the following properties of the basis functions:

1. Each $V_i(\eta)$ and $S_i(\eta)$ is continuous together with its derivative in the domain $[j-1, j]$ with degree of smoothness, $\nu = 2$.
2. Each V_i and S_i is a cubic (i.e. order $k = 4$) polynomial in each subinterval, and they vanish outside the interval $[\eta_{i-1}, \eta_{i+1}]$.
3. $$\begin{matrix} V_i(\eta_j) = \delta_{ij}, & V_i'(\eta_j) = 0 \\ S_i(\eta_j) = 0\ , & S_i'(\eta_j) = \delta_{ij} \end{matrix}, \qquad 1 \leq i, j \leq l+1$$

We now seek an approximate solution of equations (8b), (11), (12), (15) and (16) in terms of the cubic Hermite spline basis functions as

$$\phi(\eta, t) = \sum_{i=1}^{l+1} [\phi_i(t) V_i(\eta) + \phi_i'(t) S_i(\eta)] \tag{18a}$$

where the coefficients of expansion $\{\phi_i(t), \phi_i'(t)\}$, as follows from property 3 of hermite splines, are, respectively, the unknown values of the function $\phi(\eta)$ and its spatial derivative at the break point η_i with $1 \leq i \leq l+1$; i.e. $\phi_i(t) = \phi(\eta_i, t)$, $\phi_i'(t) = \phi'(\eta_i, t)$. In view of property 2 of these splines, expansion (18a) becomes

$$\phi(\eta, t) = \sum_{i=j}^{j+1} [\phi_i(t) V_i(\eta) + \phi_i'(t) S_i(\eta)] \quad \text{for} \quad \eta \in (\eta_j, \eta_{j+1}), \quad \text{with} \quad 1 \leq j \leq l \tag{18b}$$

The use of expansion (18) in equations (8b) gives

$$\begin{aligned} &\sum_{i=j}^{j+1} [\dot{\phi}_i^{(m)}(t) V_i(\eta^m) + \dot{\phi}_i'^{(m)}(t) S_i(\eta^m)] \\ &= -\frac{1}{X_m - X_{m-1}} Pe_r \sum_{i=j}^{j+1} [\phi_i^{(m)}(t) V_i'(\eta^m) + \phi_i'^{(m)}(t) S_i'(\eta^m)] \\ &\quad + \left(\frac{1}{X_m - X_{m-1}}\right)^2 \sum_{i=j}^{j+1} [\phi_i^{(m)}(t) V_i''(\eta^m) + \phi_i'^{(m)}(t) S_i''(\eta^m)] \end{aligned} \tag{19}$$

which holds for $\eta \in (\eta_j, \eta_{j+1})$ with $1 \leq j \leq l_m$ and $1 \leq m \leq N$, superscript or subscript m denotes the region number. The use of expansion (18) and property 3 of the basis functions in equation (6b) yields

$$\phi_1^{(1)}(t \geq 0) = 1, \qquad \phi_q^{(1)}(0) = 0, \qquad 2 \leq q \leq l_m + 1 \tag{20a}$$

$$\phi_i^{(m)}(0) = 0, \qquad 2 \leq m \leq N, \qquad 1 \leq i \leq l_m + 1 \tag{20b}$$

The initial values of the coefficients representing derivatives at the breakpoints are determined by fitting the profile given by equations (6b) and (6c). The use of expansion (18) and property 3 in equations (11), (12), (15) and (16) provide, respectively,

$$\dot{\phi}_1^{(1)}(t) = 0 \tag{21}$$

$$\dot{\phi}_{l_m+1}^{(m)}(t) - \dot{\phi}_1^{(m+1)}(t) = 0, \qquad 1 \leq m \leq N \tag{22}$$

$$\dot{\phi}_{l_m+1}'^{(m)}(t) - \frac{X_m - X_{m-1}}{X_{m+1} - X_m} \dot{\phi}_1'^{(m+1)}(t) = \frac{\dot{X}_m - \dot{X}_{m-1}}{X_{m+1} - X_m} \phi_1'(t) - \frac{\dot{X}_{m+1} - \dot{X}_m}{X_{m+1} - X_m} \phi_{l_m+1}' \tag{23}$$

$$\dot{\phi}_{l_N+1}'^{(N)}(t) = 0 \tag{24}$$

Equations (21)–(24) constitute $2N$ equations for an N region problem. The remaining unknown coefficients are determined by requiring equation (19) to be satisfied at a number of points in the field, called the collocation points equal to the $n-2N$, where n is the total number of unknown coefficients given by

$$n=2\sum_{m=1}^{N}(l_m+1)=2N+2\sum_{m=1}^{N}l_m \tag{25}$$

In view of approximation theory[15,16] Gauss–Legendre quadrature points of order 2 are chosen as the collocation points in each subinterval (η_j^m, η_{j+1}^m):

$$\sigma_{j,q}^m=\tfrac{1}{2}(\eta_j^m+\eta_{j+1}^m)+(-1)^q\frac{\eta_{j+1}^m-\eta_j^m}{2\sqrt{3}}, \qquad 1\leq j\leq l_m, \qquad 1\leq q\leq 2 \tag{26}$$

Evaluating equation (19) at the above mentioned collocation points leads to the following set of non-linear ordinary differential equations:

$$\begin{aligned}\sum_{i=j}^{j+1}[\dot{\phi}_i^{(m)}V_i(\sigma_{j,q}^m)+\dot{\phi}_i'^{(m)}S_i(\sigma_{j,q}^m)]\\ =-\frac{1}{X_m-X_{m-1}}Pe_r\sum_{i=j}^{j+1}[\phi_j^{(m)}V_i'(\sigma_{j,q}^m)+\phi_i'^{(m)}S_i'(\sigma_{j,q}^m)]\\ +\frac{1}{(X_m-X_{m-1})^2}\sum_{i=j}^{j+1}[\phi_j^{(m)}V_j''(\sigma_{j,q}^m)+\phi_j'^{(m)}S_i''(\sigma_{j,q}^m)]\end{aligned} \tag{27}$$

which holds for

$$\sigma_{j,q}^m\in(\eta_j^m, \eta_{j+1}^m), \qquad 1\leq j\leq l_m, \qquad 1\leq q\leq 2, \qquad 1\leq m\leq N$$

In addition, since we need a prescription for the motion of the moving boundaries of various regions, let us assume the following functional form for the motion of these boundaries

$$\dot{X}_m=f_m(Pe, \phi^{(m)}(X_m), t), \qquad 1\leq m\leq N \tag{28a}$$

where function f is a known function. In the present problem, we have chosen

$$\dot{X}_m=Pe, \qquad 1\leq m\leq N \tag{28b}$$

Equations (21–24), (27) and (28) can be compactly written in the following functional form:

$$\mathbf{A}(\mathbf{Y}, t)\dot{\mathbf{Y}}=\mathbf{G}(\mathbf{Y}, t) \tag{29}$$

where $\mathbf{Y}$ is an $(n+N)$ dimensional vector of n unknown coefficients and N co-ordinates of the moving boundaries. The coefficient matrix $\mathbf{A}(\mathbf{Y}, t)$ is a banded matrix of bandwidth equal to $2k-1$.

COMPUTATIONAL DETAILS

The solution of equation (29) is obtained by the standard library routine LSODI,[12] which solves the initial value problem with a banded coefficient matrix. It obtains the solution of a linearly implicit system of first order ODEs by Gear's multistep 'time-differencing' algorithm. Because of this algorithm, the only way one can control the time step is by varying allowable error tolerances for the integrator. This in turn implies that one cannot preselect a value of the time step or a value of the Courant number. The only other parameter that we have direct control of is the Peclet number $Pe=UL/D$ or the mesh Peclet number, $P_\Delta=U\Delta x^*/D$ which characterizes the original problem defined by equations (6) and (7) in

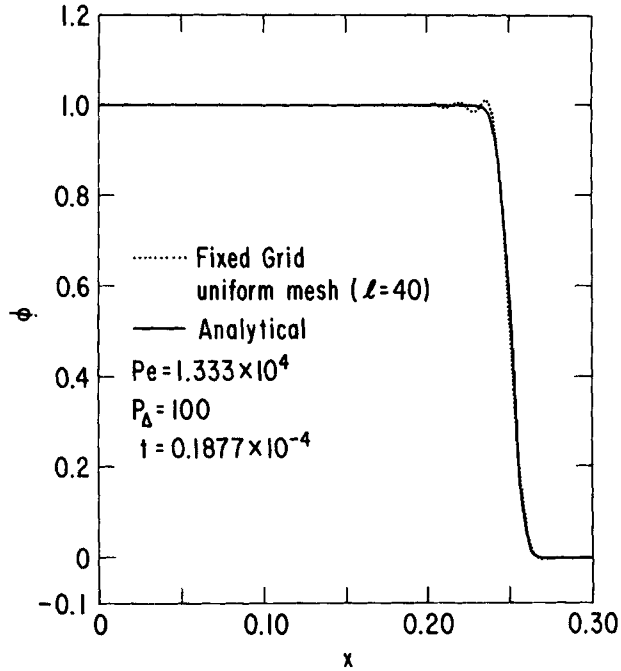

Figure 2. A comparison of fixed grid solution with analytical solution at $P_\Delta = 100$

the fixed co-ordinate system. We may note that equations (8b) reduces to equation (7) if we set $\dot{X}_j = 0$ for $1 \leqslant j \leqslant N = 1$.

The characteristics of equation (8b) for a fixed mesh are studied by varying P_Δ over a wide range. Figure 2 shows a plot of ϕ with $P_\Delta \cong 100$ with the solution obtained for the case of a fixed grid, uniform mesh distribution using multistep time-differencing. The solution for ϕ is compared with the analytical solution. As the Figure shows, there appears to be very little noise or oscillations in the computed solution; for all practical purposes this noise can be neglected. This is distinctly different from the results obtained by O'Neill and Lynch[8] who found with the finite-element method using single-step time differencing, that the solution contained significant oscillations at $P_\Delta = 100$. In fact the solution obtained by Pinder and Shapiro[9] with the same collocation method but using single-step time differencing contained oscillations even at $P_\Delta = 54$. It then appears that multistep time differencing with appropriate selection of time steps considerably improves the stability characteristics of the collocation method for the convective-diffusion problem. The extra accuracy obtainable by the use of multistep time differencing as incorporated in LSODI is not without extra cost. The ODE package requires for solution of 82 ODEs ($l = 40$) an extra storage (in addition to what may be required for single-step time differencing) of about 15 Kbytes for various auxiliary arrays used for working storage. In addition, the time steps by the LSODI integrator are significantly smaller than the time steps used in the Crank–Nicholson scheme. For example, in the above computations for $P_\Delta \simeq 100$, and the relative local time truncation error, $\varepsilon = 10^{-4}$ the Courant number ranged from 0·002 to 0·126 with an average value of about 0·098 (for a total number of steps = 342) as against a value of 0·369 used by Pinder and Shapiro[9] in their calculations at $P_\Delta \simeq 54$. The total CPU time on an IBM 3033 computer system was about 25 s in a time sharing environment.

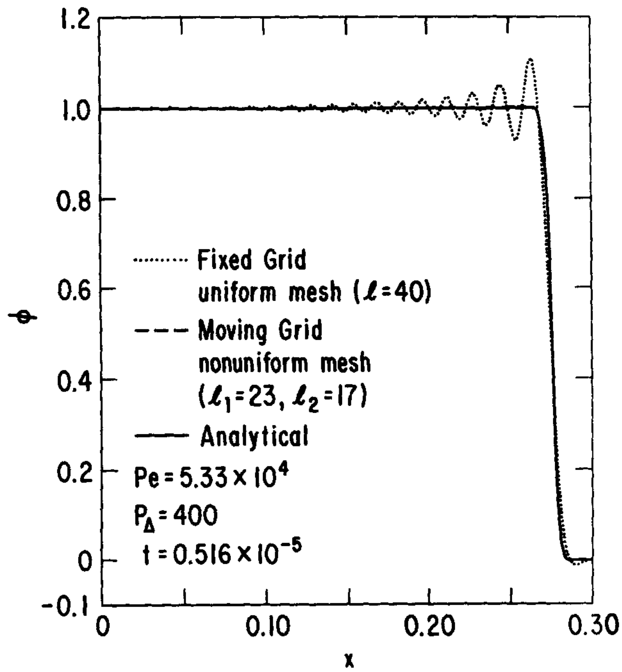

Figure 3. A comparison of fixed and moving grid solutions with analytical solution at $P_\Delta = 400$

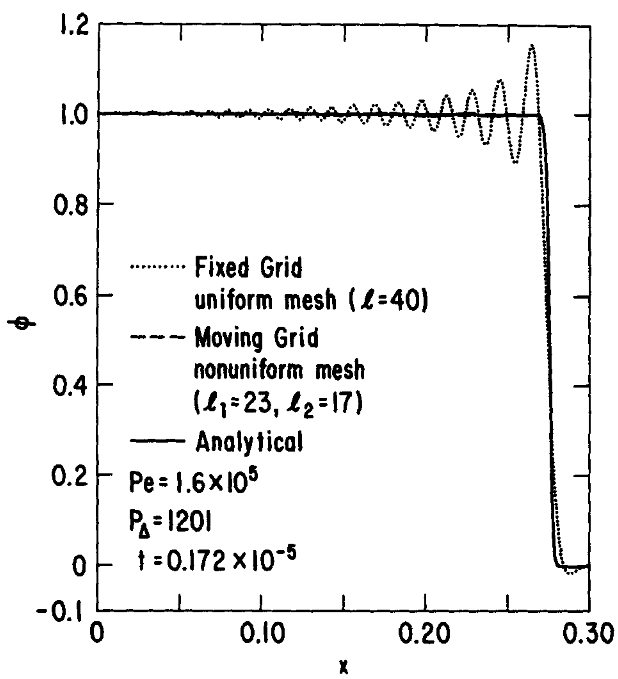

Figure 4. A comparison of fixed and moving grid solutions with analytical solution at $P_\Delta = 1201$

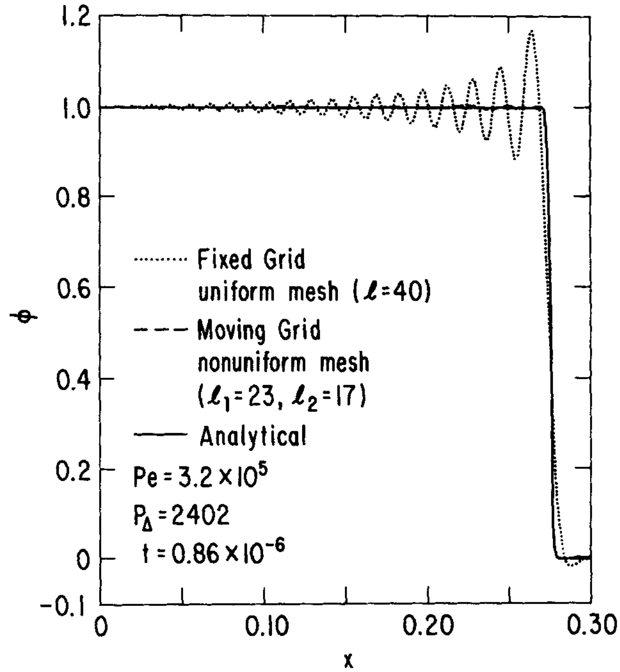

Figure 5. A comparison of fixed and moving grid solutions with analytical solution at $P_\Delta = 2402$

However, as P_Δ was increased to a value of 400, the solution for the fixed grid began to display oscillations, as can be seen in Figure 3. Also shown in this Figure is the solution obtained with moving co-ordinates having the same number of mesh points as the fixed grid case; however in the latter case, N was chosen equal to 2 and the mesh was distributed with higher concentration about the right/left boundary of the first/second region. The velocities of the moving boundaries were chosen equal to the fluid velocity U. The initial length of the first region was chosen such that the front was almost always near the left boundary of region 2 and therefore allowed the higher concentration of nodes to be placed right about the front. These features, together with the fact that the value of the characteristic Peclet number is reduced considerably by having moving co-ordinates, lead to an oscillation-free solution, as can be seen in Figure 3. As expected, the oscillations should increase in magnitude as P_Δ takes on high values. For example, with P_Δ having values 1201 and 2402, Figures 4 and 5, respectively, show considerable deterioration of the solution with the fixed grid collocation method. However, the moving mesh method with the same number of meshes but a non-uniform distribution, once again produces an oscillation-free solution which agrees extremely well with the analytical solution for both values of P_Δ.

CONCLUSIONS

The computational results presented for a very wide range of values for the mesh Peclet number clearly demonstrate the distinct advantages of this moving mesh collocation method over the fixed grid method. The moving mesh method takes advantage of the inherent high accuracy available from the use of the collocation method with the optimum choice of collocation points and multistep time differencing. The traditional fixed grid collocation method has been used extensively for the solution of non-linear problems in which diffusion or radiation effects dominate (see for example references 1–6). It is therefore expected that with the moving mesh collocation method one should be able to deal with any non-linear problem in which convection effects dominate. In addition, the solution of two-phase flow problems which offer a considerable challenge to traditional low-order finite-difference methods based on donor-cell type schemes, should be well suited for the moving mesh collocation method. Although no attempt has been made by the authors to extend these calculations to two-dimensional problems, Jensen[17] has performed such a calculation with very satisfactory results.

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